TRINITY COLLEGE

Year 12 Methods 3,4
Test 5 2021

Calculator Assumed
Normal Distribution

STUDENT'S NAME SOLUTIONS

DATE: Friday 13 August **TIME:** 40 minutes **MARKS**: 38

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser
Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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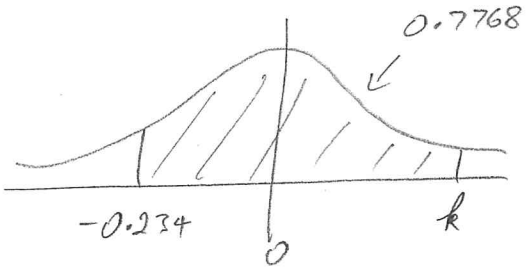
1. (4 marks)

(a) Given Z is the standard normal variable, determine k if $P(-0.234 \le Z < k) = 0.7768$ [2]

NO SOLUTION



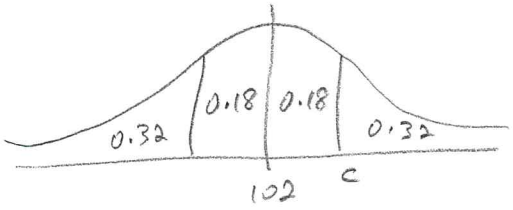
(b) Given $X \sim N(102,9)$, determine k if $P(102-k \le X \le 102+k) = 0.36$ [2]

$P(x > c) = 0.32$

$c = 103.4$

$k = 103.4 - 102$

$= 1.4$

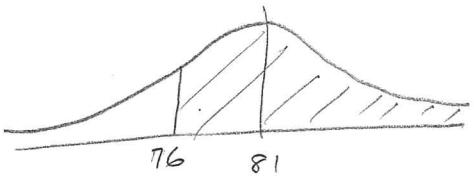


2. (20 marks)

A rock lobster fishing boat is pulling up pots at the back of Rottnest for the catch on a particularly successful day. Legal size rock lobsters must measure at least 76 mm along their shell (carapace). Past data indicates a mean length of 81 mm and standard deviation of 9 mm for rock lobsters in this area.

(a) Calculate the probability that a randomly chosen rock lobster from a pot is a legal size. [2]

$P(x \geq 76) = 0.7107$



(b) For a day when 278 rock lobsters are pulled up in pots, determine the expected number that are of a legal size. [1]

0.7107×278

$= 197.8$

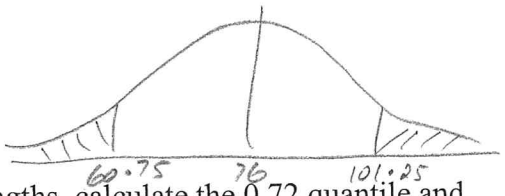
(c) Calculate the probability a randomly chosen rock lobster from a pot will have a carapace length more than one quarter of the variance from the mean. [2]

$VAR = 81$

$\frac{1}{4} VAR = 20.25$

$1 - P(60.75 \geq x \geq 101.25)$

$= 0.0244$

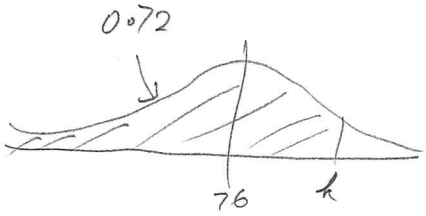


(d) For the distribution of rock lobster carapace lengths, calculate the 0.72 quantile and explain what this means. [3]

$P(x < k) = 0.72$

$k = 86.2$

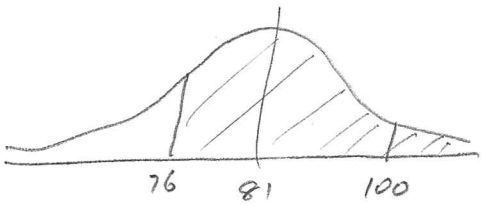
72% OF ROCK LOBSTERS HAVE CARAPACE LENGTH NO LONGER THAN 86.2 mm



(e) A rock lobster can be seen to be a legal size. What is the probability it will recorded as a jumbo? Jumbo rock lobsters must have a carapace length of at least 100 mm. [3]

$$\frac{P(X \geq 100)}{P(X \geq 76)} = \frac{0.0174}{0.7107}$$

$$= 0.0245$$



(e) 20 rock lobsters are pulled up in one pot. What is the probability that no more than 2 of them are not a legal size [3]

$Y \sim N(20, 1-0.7107)$

$b(0 \leq Y \leq 2, 20, 0.2893)$

$= 0.0439$

(f) In measuring rock lobsters at random from a pot, what is the probability that the second rock lobster not of legal size is the twelfth rock lobster measured. [3]

$b(1, 11, 0.2893) \times 0.2893$

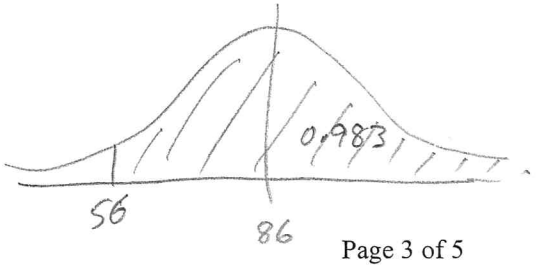
$= 0.1046 \times 0.2893$

$= 0.0303$

(g) Rock lobsters caught at the Abrolhos Islands have a mean carapace length of 86 mm. It is known that 98.3% of these rock lobsters measure over 56 mm. What is the standard deviation of carapace length of these rock lobsters? [3]

$P(X > 56) = 0.983$

$\sigma = 14.2$ mm



3. (14 marks)

A company manufactures shoelaces for joggers (shoes) which claim to be 45 cm long. The shoelaces are normally distributed with a mean of 44.9 cm and a standard deviation of 3 mm.

(a) What proportion of the shoelaces are the actual claimed length of 45 cm? [1]

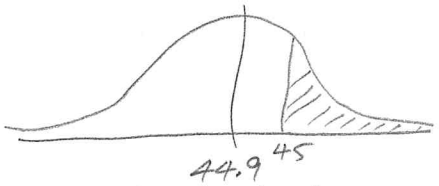
0

(b) Josh is interested in purchasing shoelaces for his joggers. In order to tie his shoelaces with ease, they must be at least 45 cm long.

(i) Josh purchases a packet of 2 shoelaces from this company. Determine the probability that they both suit his requirements. [3]

$P(x \geq 45) = 0.3645$

$0.3645^2 = 0.1365$



(ii) What is the minimum number of shoelaces needed to be purchased so that there is a 95% chance that at least two shoelaces suit Josh's requirements? [3]

$P(0) + P(1) = 0.05$

$\binom{n}{0} 0.3645^0 \times 0.6355^n + \binom{n}{1} 0.3645^1 \times 0.6355^{n-1} = 0.05$

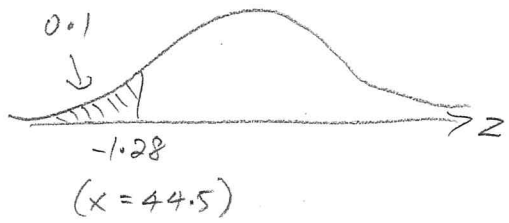
$0.6355^n + n \times 0.3645 \times 0.6355^{n-1} = 0.05$

$n \approx 11$

(c) The company also manufactures multi coloured shoelaces. A recent production run showed that 10% of these laces were less than 44.5 cm and 15% were greater than 45.6 cm. Show the mean and standard deviation of the multi coloured laces are 45.1 cm and 4.7 mm respectively. [4]

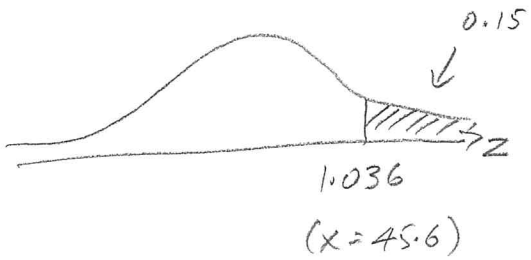
$$z = \frac{x - \mu}{\sigma}$$

$$-1.28 = \frac{44.5 - \mu}{\sigma}$$



$$z = \frac{x - \mu}{\sigma}$$

$$1.036 = \frac{45.6 - \mu}{\sigma}$$



$$\mu = 45.1$$

$$\sigma = 4.7$$

(d) Determine the value of k such that the probability the laces are longer than k cm is the same for the laces of both the joggers and the multi coloured laces. [3]

$$z = \frac{x - \mu}{\sigma}$$

$$\frac{k - 45.1}{0.47} = \frac{k - 44.9}{0.3}$$

$$k = 44.5$$